Appendix I
to
Misprescription and misuse of one-tailed tests, by C.M. Lombardi & S.H. Hurlbert

Advice given by 52 statistics books on when one-tailed tests are appropriate. See text for discussion and explanation.

Book	One-tailed tests are appropriate ...

Sombreros Blancos: Reasonable Advice (n = 12)

Edwards (1967)
- if the experimenter "in advance of the experiment itself, [has] a *practical* reason for deciding that he is interested only in" a difference in a particular direction (p.227)

Welkowitz, Ewen & Cohen (1971, 1991)
- rarely or never; "in almost all situations in the behavioral sciences, extreme results in either direction are of interest" (p. 128, p.150).

Fleiss (1981)
- "only when the investigator is not interested in a difference in the reverse direction from that hypothesized . . . Such an instance is assuredly rare" (p. 29)

Cochran (1983)
- "only when we know in advance what sign δ must have if it does not equal zero" (p.21) [This equates to "never"!]

Fleiss (1986)
- "When the consequences of equality and inferiority [e.g. of an experimental drug treatment relative to a control treatment] are the same, *and only then*" (p. 94)

Campbell (1989)
- when we "have sufficient prior knowledge of the populations to be sure that the sample deviation in one sense can be due only to chance...[If a result in the unexpected direction occurs] we will be confident that this is a chance effect and will not wish to perform a significance test .... one-sided tests are justified only rather rarely." (p.87)

Altman (1991)
- only "in rare cases [where] it is reasonable to consider that a real difference can occur in only one direction, so that an observed difference in the opposite direction must be due to chance ... one-tailed tests are rarely appropriate" (p. 171)

Winer et al. (1991)
- if "the experimenter is interested in rejecting H_O only when the alternative hypothesis is one having a specified direction" (p.44)

Schulman (1992)
- "If we are only interested in departures from the null hypothesis in a single direction...We should...perform a two-sided test when[ever] it would be worth uncovering a departure from the hypothesized value [e.g. $\delta=0$] in either direction" (p.80)

Helsel & Hirsch (1992)
- "If it can...be stated prior to looking at any data that departures from H_O in only one direction are of interest" (p. 106)

Bart et al. (1998)
- only "if one is certain that the true difference between the parameter and its value specified under the null hypothesis cannot be positive (or that it cannot be negative). This assumption is rarely justified in behavioral ecology." (p.57)

Hawkins (2005)
- when there is "an exceptional reason.....Just being interested in one direction is not enough .... I recommend avoiding one-tailed tests like the plague" (p.90).

Sombreros Grises: Vague, Inconsistent, or Mixed Advice (n = 25)

McNemar (1955)
- when "we are concerned only with results in one direction" (p. 62)
- when "one predicts on the basis of theory or previous observation the outcome of an experiment" (p. 63)

McNemar (1969)
- when "the scientific hypothesis being tested [or] ... the practical decision to be made demands that we be concerned with chance deviations in just one direction ..." (p. 64)
- "whenever the outcome of an experiment is predicted on the basis of theory or previous observation" (p. 64)

Armitage (1971)
- "If, for some reason, we decided that we were interested in possible [differences] ... only in one particular direction;"
- "only if it is quite certain that departures in one particular direction will always be ascribed to chance, and therefore regarded as non-significant however large they are. This situation rarely arises in practice, and it will be safe to assume that significance tests should almost always be two-sided" (p.104)

Hays & Winkler (1971)
- if one is "looking for a directional difference between populations" (p.414)
- when it is "clear that the only alternative of logical or practical consequence must lie in a certain direction" (p.416);
- when "[w]e have not the slightest practical interest in" an effect in the opposite direction (p.417)
- [identical language is used in Hays 1981, p.260-261, and Hays 1988, p.276-277]

Edwards (1972)
- "If we have [a] ... prior hypothesis about the direction of the difference" (p. 87)
- "if we are interested in [both] the absolute magnitude of the difference ... and ... the direction of the difference" (p. 87)
- "only if we have no interest whatsoever" in an effect in the opposite direction (p. 88)

Box, Hunter & Hunter (1978)
- [make heavy use of 1-tailed examples from applied research but never state a general criterion for determining when they are appropriate (e.g. p.108)]

Kurtz & Mayo (1979)
- rarely; only when "your intructor gives you reasons for doing" so! (p.140)

Conover (1980)
- [gives many 1-tailed and 2-tailed examples, but never states a general criterion for deciding which type of test to use]

Steel & Torrie (1980); Steel, Torrie & Dickey (1997)
- [discuss and demonstrate 1-tailed tests in a few places but never state a general criterion for determining when they are appropriate]

Sokal & Rohlf (1981, 1995)
- when "we know from past experience ... [or] we have reason to believe" that the difference will be in a particular direction, or "we are interested in" only one direction (pp.79, 168-169)

Dixon & Massey (1983)
- in "situations . . . in which only large (or small) alternative values are of interest" (p.91)

Wonnacott & Wonnacott (1985)
- "when there is a one-sided claim to be made such as 'more than', 'less than'..." (p.283)

Daniel (1987)
- "depend[ing] on the nature of the question being asked by the researcher" (p.198)

Bland (1987)
- well, it "depends on the field in which the testing is usually done. In Biological Science, treatments seldom have only one effect and relationships between variables are usually complex. Two-sided tests would seem to be generally preferable" (p.152)
- [gives example where a 1-tailed test is used because a particular result is predicted, p.152]

Siegel & Castellan (1988)
- "if the theory predicts the *direction* of the difference" (p. 8; see also pp. 41, 42, 60, 81, 89, 90, 92)
- "if the researcher were only interested in determining whether" the difference was in a particular direction (p. 14)
- when there are "*a priori* notions about" the direction of the difference (p. 309)

Moore & McCabe (1989)
- when "we are interested in detecting only" a difference in a particular direction;
- when we have "hopes or suspicions" concerning the direction of the difference (p.464);
- when we "have a specific direction firmly in mind in advance" (p.465)

Samuels (1989)
- "when only one direction of deviation . . . is believed to be plausible;"
- "when deviation in both directions is possible, but only one direction is of primary interest" (p. 222)

Snedecor & Cochran (1989)
- when "the investigator knows enough about the circumstances to be certain that" a difference can occur only in one direction;
- "where the test criterion can change in only one direction when the null hypothesis is false. The χ^2 goodness of fit test ... is one example"
- "when the investigator is simply not interested in distinguishing between the null hypothesis and some of the alternatives" (p.67)

Neter, Wasserman & Kutner (1990)
- [some 1-tailed examples are presented, but no general criterion for use of such tests is specified]

Freedman, Pisani & Purves (1991, 1998)
- "when the alternative hypothesis says" the result is in a particular direction (p.551)….and "investigators rport the *P*-value instead of just comparing *P* to 5% or 1% (p.578)

Bolton (1997)
- "in certain circumstances ....[e.g.] if evidence were available to show that a new [production] process could not reduce the potency [of a tablet product], a one-sided test would be acceptable. To have such evidence and convince others (particularly regulatory agencies) of its validity is not always an easy task. Also, from a scientific point of view, two-sided tests are desirable because significant results in both positive and negative directions are usually of interest" (p.143)

Welkowitz, Ewen & Cohen (1999)
- [This edition, in contrast with earlier ones, omits all advice on 1- versus 2-tailed tests]

Quinn & Keough (2002)
- if "[w]e might be interested in whether one mean is bigger than the other mean but not the other way. For example, we might expect increased density of organisms to induce competition and reduce their growth rate, and we can think of no mechanism whereby the organisms at the higher density would increase their growth....[but if we find them] we are obliged to ignore differences in the opposite direction...Is this unrealistic, expecting a biologist to ignore what might be an important effect just because it was in the opposite direction to that expected? This might seem like an argument against one-tailed tests, avoiding the problem by never ruling out interest in effects in both directions...." (p.37)

Gotelli & Ellison (2004)
- depending on how you would "interpret an extremely large or an extremely small value of the response variable." (p. 114)

Zar (2004)
- "when our interest lies only" in a difference in a particular direction (p.96; see also pp. 86, 125)
- "If we had reason to ask whether some treatment" had an effect in a particular direction (p.578)
- if there is a "reason, *a priori*, to hypothesize that a change ....would be in one specified direction" (p. 579)

Sombreros Negros: Bad Advice (n = 15)

Edwards (1954)

- when there is "a theoretical basis for predicting, in advance, "that the difference will be in a particular direction (p.258);
- "if the experimenter has an *a priori* hypothesis concerning the outcome of the experiment" (p.290)

Siegel (1956)

- "when we have predicted in advance" the direction of the difference (p.39; see also pp. 7, 40, 42, 54, 69, 77, 78, 81, 86)

Ferguson (1971)

- if concern is with the sign of the difference as well as with its absolute magnitude;
- if "we wish to make a decision about the direction of the difference" (p.150);
- "where the direction of the differences is of substantial interest... it is the opinion of this writer that directional tests should be used more frequently." (p.151)

Kendall & Stuart (1979)

- "when we have prior knowledge about" the likely direction of the difference (p.195)

Wonnacott & Wonnacott (1981)

- if "on theoretical grounds, it may be concluded a priori "that the result can only be in one direction (p.39)
- when there is "a prior theoretical reason for expecting [X] to affect Y in a specific direction." (p.90)

Kachigan (1986)

- when we are "absolutely certain, usually on logical grounds, that ... [a difference in the opposite direction] has a zero probability of occurrence" (p.161)

Kirk (1982)

- "whenever the experimenter makes a *directional* prediction concerning the phenomenon of interest"; [if] we ... have sufficient information to make a directional prediction"; when the experimenter has a "hunch" that the difference will be in a particular direction (p.34)

Marks (1982)

- "only if the researcher is interested intuitively in showing a difference in one direction" (p.124)

Glass & Hopkins (1984)

- "when the investigator believes" (p. 214) or "where expectations are strong" (p. 304) that the result will be in a particular direction;
- "when [the tests] are properly guided by sound theory or previous research" (p. 303)

Sachs (1984)

- when "a *substantiated hypothesis* allows us to make certain predictions about the sign of the expected difference;"
- "If the sign of a presumable difference of the two parameters . . . is known" beforehand (p. 125)

Martin & Bateson (1986; 2007)

- "If a prior prediction is made about the *direction* of an effect ... If ... existing knowledge or theory firmly predicts the direction of the difference" (p.119; p. 157-158)

Darlington & Carlson (1987)

- if "the investigator may hope or expect that the result will deviate from the null hypothesis in a particular direction" (p.206);
- if "the scientist has a specific theory that predicts" the direction of the difference (p.207);
- if "results in the opposite direction would make no theoretical sense at all" (p.209)

Robertson, Wright & Dykstra (1988)

- if one makes "the assumption that the first mean is larger than the second" (p.3)

Underwood (1997)

- when "the logical processes used by the experimenter" so dictate (p.64)
- when it is "expected that" the difference will be in a particular direction (p.137)

Kline (2004)

- "If there are a priori reasons to expect a directional effect…" (p.38)

BOOKS CITED

Altman D. G. (1991) *Practical Statistics for Medical Research*. Chapman and Hall, New York.

Armitage P. (1971) *Statistical Methods in Medical Research*. Blackwell Scientific Publications, Oxford.

Bart J., Fligner M. A. & Notz W. I. (1998) *Sampling and Statistical Methods for Behavioral Ecologists*. Cambridge University Press, Cambridge.

Bland M. (1987) *An Introduction to Medical Statistics*. Oxford University Press, Oxford.

Bolton S. (1997) *Pharmaceutical Statistics: Practical and Clinical Applications*, 3d edn. Dekker, New York.

Box G. E. P., Hunter W. G. & Hunter J. S. (1978) *Statistics for Experimenters*. John Wiley and Sons, New York.

Campbell R. C. (1989) *Statistics for Biologists*, 3d edn. Cambridge University Press, Cambridge.

Cochran W. G. (1983) *Planning and Analysis of Observational Studies*. Wiley, New York.

Conover W. J. (1980) *Practical Nonparametric Statistics*. Wiley, New York.

Daniel W.W. (1987) *Biostatistics: A Foundation for Analysis in the Health Sciences*, 4th edn. John Wiley and Sons, New York.

Darlington R. B. & Carlson P. M. (1987) *Behavioral Statistics*. The Free Press, New York.

Dixon W.J. & Massey, F.J. Jr. (1983) *Introduction to Statistical Analysis*, 4th edn. McGraw-Hill Book Company, New York.

Edwards A. L. (1954) *Statistical Methods for the Behavioral Sciences*. Rinehart, New York.

Edwards A. L. (1967) *Statistical Methods*, 2d edn. Holt, Rinehart, and Winston, Inc., New York.

Edwards A. L. (1972) *Experimental Design in Psychological Research*, 4th edn. Holt, Rinehart, and Winston, New York.

Ferguson G. A. (1971) *Statistical Analysis in Psychology and Education*, 3rd edn. McGraw-Hill Book Company, New York.

Fleiss J. L. (1981) *Statistical Methods for Rates and Proportions*, 2nd edn. Wiley, New York.

Fleiss J. L. (1986) *The Design and Analysis of Clinical Experiments*. Wiley and Sons, New York.

Freedman D., Pisani R., Purves R. & Adhikari A. (1991, 1998) *Statistics*, 2d, 3d edn. Norton, New York.

Glass G. V. & Hopkins K. D. (1984) *Statistical Methods in Education and Psychology*, 2nd edn. Englewood Cliffs, Prentice-Hall, New Jersey.

Gotelli N.J. & Ellison A.M. (2004) *A Primer of Ecological Statistics*. Sinauer, Sunderland, Massachusetts.

Hawkins D. (2005) *Biomeasurement*. Oxford Univ. Press, New York.

Hays W. L. & Winkler R. L. (1971) *Statistics, Probability, Inference and Decision*. Holt, Rinehart, & Winston, Inc., New York.

Helsel D. R. & Hirsch R. M. (1992) *Statistical Methods in Water Resources*. Elsevier, Amsterdam.

Hollander M. & Wolfe D. A. (1973) *Nonparametric statistical methods*. Wiley, New York.

Kachigan S. K. (1986) *Statistical Analysis*, 2nd ed. Radius Press, New York.

Kendall M. & Stuart A. (1979) *The Advanced Theory of Statistics*, *vol. 2: Inference and Relationship,* 4th ed. Griffin, London.

Kirk R. E. (1982) *Experimental Design*, 2nd ed. Brooks/Cole Publishing Company, Pacific Grove, California.

Kline R. B. (2004) *Beyond Significance Testing: Reforming Data Analysis Methods in Behavioral Research*. American Psychological Association, Washington DC. 325 pp.

Kurtz A. K. & Mayo S. T. (1979) *Statistical Methods in Education and Psychology*. Springer-Verlag, New York.

Marks R.G. (1982) *Designing a Research Project: The Basics of Biomedical Research Methodology*. Lifetime Learning Publications, Belmont, California.

Martin P. & Bateson P. (1993, 2007) *Measuring Behaviour: An Introductory Guide*, 2nd, 3d edns. Cambridge University Press, Cambridge.

McNemar Q. (1955) *Psychological Statistics*, 2d edn. Wiley, New York.

McNemar Q. (1969) *Psychological Statistics*, 4th edn. Wiley, New York.

Meddis R. (1984) *Statistics Using Ranks*. Blackwell Scientific Publications, Oxford.

Moore D.S. & McCabe G.P. (1989) *Introduction to the Practice of Statistics*. W.H. Freeman and Company, New York.

Neter J., Wasserman W. & Kutner M. H. (1990) *Applied Linear Statistical Models*, 3rd edn. IRWIN, Homewood, Illinois.

Quinn G. P. & Keough M. J. (2002) *Experimental Design and Data Analysis for Biologists*. Cambridge University Press, New York.

Robertson T., Wright F. T. & Dykstra R. L. (1989) *Order Restricted Statistical Inference.* Wiley, London.

Sachs L. (1984) *Applied Statistics*, 2d edn. Springer, New York.

Samuels M. L. (1989) *Statistics for the Life Sciences.* Dellen Publ. Co., San Francisco.

Schulman R. S. (1992) *Statistics in Plain English*. Van Nostrand Reinhold, New York.

Siegel S. (1956) *Nonparametric Statistics for the Behavioral Sciences*. McGraw-Hill, New York.

Siegel S. & Castellan, N. J. Jr. (1988) *Nonparametric Statistics for the Behavioral Sciences*, 2nd edn. McGraw-Hill, New York.

Snedecor G. W. & Cochran W. G. (1989) *Statistical Methods*, 8th edn. Iowa State University, Ames.

Sokal R. R. & Rohlf F. J. (1981, 1995) *Biometry*, 1st, 2nd, 3d edn. Freeman, San Francisco.

Steel R. G. D. & Torrie J. H. (1980) *Principles and Procedures of Statistics*, 2nd edn. McGraw-Hill, New York.

Steel R.G.D., Torrie J.H. & Dickey D.A. (1997) *Principles and Procedures of Statistics*, 3d edn. McGraw-Hill, New York.

Underwood A. J. (1997) *Experiments in Ecology*. Blackwell, London.

Welkowitz J., Ewen R. B. & Cohen J. (1971, 1991, 1999) *Introductory Statistics for the Behavioral Sciences*, 1st, 4th, 5th edn. Harcourt Brace Jovanovich, New York.

Winer B. J., Brown D. R. & Michels K. M. (1991) *Statistical Principles in Experimental Design*, 3rd edn. McGraw-Hill, New York.

Wonnacott R. J. & Wonnacott T. H, (1985) *Introductory Statistics*, 4th edn. Wiley, New York.

Wonnacott T. H. & Wonnacott R. J. (1981) *Regression: A Second Course in Statistics*. Wiley, New York.

Zar J. H. (2004) *Biostatistical Analysis*, 5th edn. Prentice-Hall, Inc., New York.